

Straight Lines

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. If A and B are two persons standing at the positions (2, -3) and (6, -5). If C is a third person who is standing between A and B such that it divides the line AB in the ratio 1:3.



Based on the above information answer the following questions.

(A) The distance between A and B is:

- (a) $\sqrt{5}$
- (b) $2\sqrt{5}$
- (c) $3\sqrt{5}$
- (d) $4\sqrt{5}$

(B) The equation of AB is:

- (a) $x+2y+4=0$
- (b) $x+2y-4=0$
- (c) $x-2y+4=0$
- (d) none of these

(C) Coordinates of points C are:

- (a) $\left(\frac{7}{2}, -3\right)$
- (b) $\left(3, \frac{7}{2}\right)$
- (c) $(3, 3)$
- (d) $\left(3, -\frac{7}{2}\right)$

(D) Distance between A and C is:



(a) $\sqrt{5}$ (b) $2\sqrt{5}$

(c) $\frac{\sqrt{5}}{2}$ (d) $\sqrt{\frac{5}{2}}$

(E) Distance between C and B is:

(a) $\frac{3\sqrt{5}}{2}$ (b) $3\sqrt{5}$

(c) $\frac{2\sqrt{5}}{3}$ (d) None of these

Ans. (A) (b) $2\sqrt{5}$

Explanation: Given positions of person A and B are as follows:

A(2,-3) and B(6,-5)

$$\begin{aligned}d &= \sqrt{(6-2)^2 + (-5+3)^2} \\&\quad \text{[using distance formula]} \\&= \sqrt{(4)^2 + (-2)^2} = \sqrt{16+4} \\&= \sqrt{20} = 2\sqrt{5}\end{aligned}$$

(B) (a) $x+2y+4=0$

Explanation: We have, A(2, -3) and B(6,-5)

$$\begin{aligned}\text{Slope, } m &= \frac{-5-(-3)}{6-2} \\&= \frac{-5+3}{4} \\&= \frac{-2}{4} = -\frac{1}{2}\end{aligned}$$

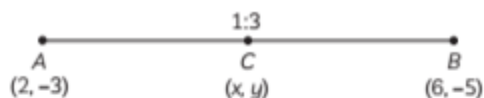
Taking point A(2, -3) = (x_1, y_1) and $m = -\frac{1}{2}$

Equation of line AB is

$$\begin{aligned}(y - (-3)) &= -\frac{1}{2}(x - 2) \\ \Rightarrow 2(y + 3) &= -(x - 2) \\ \Rightarrow 2y + 6 &= -x + 2 \\ \Rightarrow x + 2y + 4 &= 0\end{aligned}$$

(C) (d) $\left(3, -\frac{7}{2}\right)$

Explanation: Let point C divides AB in the ratio m_1 , and m_2 .



$$\begin{aligned}\text{Then, } (x, y) &= \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right) \\ &= \left(\frac{1 \times 6 + 3 \times 2}{1 + 3}, \frac{1 \times (-5) + 3 \times (-3)}{1 + 3} \right) \\ &= \left(\frac{12}{4}, \frac{-14}{4} \right) = \left(3, -\frac{7}{2} \right)\end{aligned}$$

(D)

(c) $\frac{\sqrt{5}}{2}$

Explanation: We have, A(2, -3) and

$C\left(3, -\frac{7}{2}\right)$

$$\begin{aligned}AC &= \sqrt{(3-2)^2 + \left(-\frac{7}{2} + 3\right)^2} \\ &= \sqrt{1^2 + \left(-\frac{1}{2}\right)^2} \\ &= \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}\end{aligned}$$

(E)

(a) $\frac{3\sqrt{5}}{2}$

Explanation: We have, $C\left(3, -\frac{7}{2}\right)$ and

$B(6, -5)$

$$\begin{aligned}CB &= \sqrt{(6-3)^2 + \left(-5 + \frac{7}{2}\right)^2} \\ &= \sqrt{3^2 + \left(-\frac{3}{2}\right)^2} \\ &= \sqrt{9 + \frac{9}{4}} = \frac{\sqrt{45}}{2} = \frac{3\sqrt{5}}{2}\end{aligned}$$

2. The corner points of a square plot are (1, 2), (2, 3), (3, 1) (0,-4). Plot is located in an industrial area to build a well known company showroom.



Based on the above information, answer the following questions:

- (A)** Find the distance between (1, 2) and (3, 1) and also find the slope of (1, 2) and (3, 1).
(B) Find the distance between (2, 3) and (0, -4) and also find the slope of (2, 3) and (0, -4).
(C) Determine $\angle B$ of the triangle with vertices A(-2, 1), B(2, 3) and C(-2, -4).

Ans. (A) Here, A = (1, 2) B = (3, 1)

$$AB = \sqrt{(3-1)^2 + (1-2)^2}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5}$$

Here, $x_1 = 2, x_2 = 0$

$$y_1 = 3, y_2 = -4$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4-3}{0-2}$$

$$= \frac{-7}{-2}$$

$$= \frac{7}{2}$$



(B)

Here, $A = (2, 3)$

$B = (0, -4)$

$$\begin{aligned}\text{So, } AB &= \sqrt{(0-2)^2 + (-4-3)^2} \\ &= \sqrt{4+49} \\ &= \sqrt{53}\end{aligned}$$

Here, $x_1 = 1, x_2 = 3$

$y_1 = 2, y_2 = 1$

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-2}{3-1} \\ &= \frac{-1}{2}\end{aligned}$$

(C)

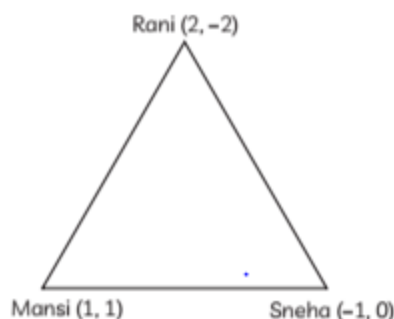
Slope of line, AB = $\frac{3-1}{2+2} = \frac{2}{4} = \frac{1}{2} = m_1$ (say)

Slope of line, BC = $\frac{-4-3}{-2-2} = \frac{7}{4} = m_2$

$$\begin{aligned}\therefore \tan B &= \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| \\ &= \left| \frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{1}{2} \cdot \frac{7}{4}} \right| \\ \angle B &= \tan^{-1} \left(\frac{2}{3} \right)\end{aligned}$$

3. Three girls Rani, Mansi, Sneha are talking to each other while maintaining a social distance due to covid-19. They are standing on vertices of a triangle, whose coordinates are given.





Based on the above information answer the following questions.

(A) The equation of lines formed by Rani and Mansi is:

- (a) $3x - y = 4$
- (b) $3x + y = 4$
- (c) $x - 3y = 4$
- (d) $x + 3y = 4$

(B) Slope of equation of line formed by Rani and Sneha is:

- (a) $\frac{2}{3}$
- (b) $\frac{-3}{2}$
- (c) $\frac{-2}{3}$
- (d) $\frac{1}{3}$

(C) The equation of median of lines through Rani is:

- (a) $5x + 4y = 2$
- (b) $5x - 4y = 2$
- (c) $4x - 5y = 1$
- (D) The equation of

(d) none of these altitude through Mansi is:

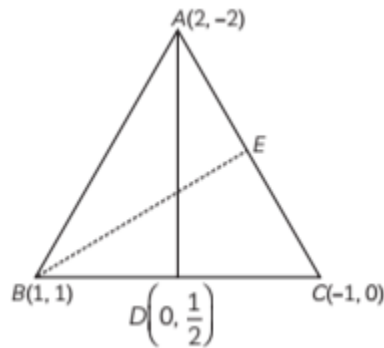
- (a) $3x - 2y = 1$
- (b) $2x + 3y = 5$
- (c) $x + 2y = 3$
- (d) none of these

(E) The equation of line passing through the Rani and parallel to line formed by Mansi and Sneha is:

- (a) $x - 2y = 4$
- (b) $x + 2y = 6$
- (c) $x - 2y = 6$
- (d) $2x + y = 4$

Ans. Let the point on Rani, Mansi and Sneha stand on a vertices of triangles be A, B, C.

$\therefore A(2, -2), B(1, 1), C(-1, 0)$



(A) (b) $3x+y=4$

Explanation: The equation of line AB is

$$y - 1 = \frac{-2-1}{2-1}(x-1)$$

$$\left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \right]$$

$$\Rightarrow y - 1 = -3x + 3$$

$$\Rightarrow 3x + y = 4$$

(B)

(c) $-\frac{2}{3}$

Explanation: Slope of equation of line AC is

$$m = \frac{0+2}{-1-2} = \frac{2}{-3} = -\frac{2}{3}$$

(C) (a) $5x + 4y = 2$

Explanation: Let D be the mid-point of BC.

$$\text{Coordinates of D are } \left(\frac{1-1}{2}, \frac{0+1}{2} \right) = \left(0, \frac{1}{2} \right)$$

$$\therefore \text{Equation of AD is } y + 2 = \frac{\frac{1}{2} + 2}{0 - 2}(x - 2)$$

$$\Rightarrow y + 2 = \frac{-5}{4}(x - 2)$$

$$\Rightarrow 4y + 8 = -5x + 10$$

$$\Rightarrow 5x + 4y = 2$$

(D) (a) $3x-2y=1$

Explanation: Slope of AC = $\frac{-2}{3}$

$\therefore$ Slope of BE = $\frac{3}{2}$ [$\because$ BE $\perp$ AC]

Equation of altitude through B is

$$y - 1 = \frac{3}{2}(x - 1)$$

$$\Rightarrow 3x - 2y = 1$$

(E) (c) $x - 2y = 6$

Explanation: Slope of line BC = $\frac{0 - 1}{-1 - 1} = \frac{1}{2}$

Equation of line passing through A and parallel to BC is

$$y + 2 = \frac{1}{2}(x - 2)$$

$$\Rightarrow 2y + 4 = x - 2$$

$$\Rightarrow x - 2y = 6$$

4. A triangular park has two of its vertices as B(-4, 1) and C(2, 11). The third vertex A is a point dividing the line joining the points (3, 1) and (6, 4) in the ratio 2:1.

(A) Find the coordinates of third vertex A.

(B) Find the equation of line passing through B and C.

(C) Find the equations of the sides of a triangle whose vertices are A(-1, 8), B(4, -2) and C(-5, -3).

Ans. (A)



Coordinates of

$$A = \left(\frac{2 \times 6 + 1 \times 3}{2 + 1}, \frac{2 \times 4 + 1 \times 1}{2 + 1} \right)$$

i. e., (5, 3)

(B) Equation of line through B(-4, 1) and C(2, 11) is

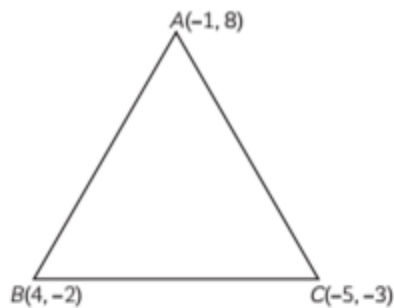
$$y - 1 = \frac{11 - 1}{2 - (-4)}(x + 4)$$

$$\Rightarrow y - 1 = \frac{5}{3}(x + 4)$$

$$\Rightarrow 3y - 3 = 5x + 20$$

$$\Rightarrow 5x - 3y + 23 = 0$$

(C) Here, we use two points form to find the equation of sides.



$$\text{Equation of AB is } y - 8 = \frac{-2 - 8}{4 - (-1)}(x + 1)$$

$$\Rightarrow 5(y - 8) + 10(x + 1) = 0$$

$$\Rightarrow 10x + 5y - 30 = 0$$

$$\Rightarrow 2x + y - 6 = 0$$

[dividing both sides by 5]

Equation of BC is

$$y + 2 = \frac{-3 - (-2)}{-5 - 4}(x - 4)$$

$$\Rightarrow -9(y + 2) + (x - 4) = 0$$

$$\Rightarrow x - 9y - 22 = 0$$

$$\text{Equation of AC is } y - 8 = \frac{-3 - 8}{-5 - (-1)}(x + 1)$$

$$\Rightarrow -4(y - 8) + 11(x + 1) = 0$$

$$\Rightarrow 11x - 4y + 43 = 0$$